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THREE-DIMENSIONAL ANALYSIS OF CRACK IN CENTRALLY PERFORATED PHOTOELASTIC CYLINDERS UNDER INTERNAL PRESSURE

C. T. Liu
AFRL/PRSM
10 E. Saturn Blvd.
Edwards AFB CA 93524-7680, U. S. A

C. W. Smith
Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, VA 24060, U. S. A

ABSTRACT

In this study, the frozen stress photoelastic method was used to investigate the three-dimensional effect on the crack growth behavior in a centrally perforated circular cylinder under internal pressure. The inner surface of the cylinder had a star shape, which consisted of six fins. The specimens were capped at the ends and pressurized internally above critical temperature after real cracks were introduced at the fin tip. After growing to a desired size, the pressure was reduced to stop crack growth and held through cooling. Two different types of cracks, part-through crack and long crack with the crack length nearly equal to the length of the cylinder, were considered. The experimental data were analyzed, and the results are discussed.

INTRODUCTION

An important engineering problem in structural design is evaluating structural integrity and reliability. It is well known that structural strength may be degraded by the presence of cracks in the material. In order to determine the severity of the crack or the service life of the structure, the failure criterion should include the crack propagation aspect of the localized failure. Since the crack growth behavior is controlled by the local stress at the crack tip, the distribution of the stress intensity factor along the crack front needs to be determined.

The application of the frozen stress photoelastic method to the study of stresses in three-dimensional specimens is not new. An excellent discourse on this subject was given by Durelli [1]. Subsequently, Francis et al. [2] studied the pressurized crack behavior in a two-dimensional models of cylindrical specimen. Smith el at. [3] conducted photoelastic experiments on nozzle corner cracks in a pressurized vessel to determine the distribution of stress intensity factor along the crack front. In 1993, a systematic study was conducted by Smith et al. [4] to determine the stress intensity factor along the front of part-through cracks with different sizes in centrally perforated cylinders. Very recently, Leblond [5] provided a three-dimensional analytical framework for use in analyzing problems with linear elastic fracture mechanics constraints when the crack configuration is known.

In this study, the frozen stress photoelastic method was used to investigate the three-dimensional effect on the crack growth behavior in a centrally perforated circular cylinder under internal pressure. The inner surface of the cylinder had a star shape, which consisted of six fins (Fig.1). The specimens were capped at the ends and pressurized internally above critical temperature after real cracks were introduced at the fin tip. After growing to a desired size, the pressure was reduced to stop crack growth and held through cooling. Two different types of cracks, a grown part-through crack and a machined long crack with the crack length nearly equal to the length of the cylinder (Fig.2), were considered. The experimental data were analyzed, and the results are discussed.

THE EXPERIMENTS

In order to obtain some insight into the three-dimensional effects on crack growth behavior under load, a series of experiments on pre-cracked centrally perforated cylindrical specimens, made of a photoelastic material, were conducted using the frozen stress method. The starter part-through crack was made by first drilling a small hole opposite the fin in which the crack was to be located, sliding a shaft with a tip blade into the hole, positioning the blade at the critical point on the fin surface, and then striking the shaft with a hammer. The starter crack then emanated from the blade tip into the materials as "a natural" or a real crack. For the long crack, the

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14. ABSTRACT
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specimen was cut in half along a plane normal to the axis of the fin tip from which the crack symmetric to load and geometry was to be placed. This crack was then machined in and extended at constant depth until near the ends of the specimen where it was turned up into the fin surface (Fig.2). Although shape of the long crack was not a semi-elliptical, the shape factor for this crack varied by < 2% from unity. The specimen with a short natural crack was capped at the ends and pressurized internally above critical temperature after the natural crack was introduced at the fin tip and grew to a desired depth after which pressure reduced for stress freezing. The long crack specimen was capped at the ends and heated above critical temperature. Then internal pressure was applied. Subsequently, the specimen was slowly cooled to room temperature and the load removed. Thin slices were removed mutually orthogonal to the crack border and its surface and analyzed photoelastically as two-dimensional models but containing the three dimensional effect. The photoelastic data were converted into stress intensity factor, SIF, using two parameter algorithms developed by Smith and Kobayashi [6].

RESULTS AND SISCUSSIONS

Photoelastic analysis of an uncracked specimen under internal pressure showed that there were two critical locations at a fin tip; one at the confluence of the edge radius of R = 1.3 mm with the radius R = 11.0 mm of the central part of the fin tip, and the other one on the fin axis itself (Fig.3). There were two positions on each fin tip where the confluence of the two above noted radii existed. A crack emanating from such a position was defined as off-axis crack (Fig.1). A crack at the other location on the fin axis was defined as a symmetric crack, which is symmetric with respect to both load and specimen geometry. Off-axis cracks however generally do not occupy the principal planes of stress or planes of symmetry in their early stages of growth. When the off-axis crack propagates, it must turn to eliminate some shear modes before becoming purely Mode I at which time it will grow readily as a symmetric crack. Experimental results revealed that symmetric cracks penetrated to the outer surface of the specimen before the off-axis cracks had grown significantly. In other words, the symmetric crack propagated much faster than the off-axis crack, which implied that the symmetric crack was more critical than the off-axis crack as far as the evaluation of the structural integrity was concerned. Therefore, it is more logical and important to investigate the growth behavior of the symmetric crack than that of the off-axis crack even though the stresses at the off-axis crack location are higher than those at the symmetric crack location.

It is well known that there are a number of existing theories, such as maximum normal stress criterion and strain energy criterion, that can be used to predict the crack growth direction. Experimental results obtained by Smith el at. [3] in their studies on nozzle corner cracks in a pressurized vessel and symmetric cracks in centrally perforated cylinders under internal pressure suggest that a crack will propagate perpendicular to the maximum principal stress direction. For the specimen geometry and the crack location considered in this study, the axis of symmetry is the trajectory perpendicular to the maximum principal stress direction. Therefore, when a symmetric crack grows in the depth direction, it will propagate in the plane of symmetry and under pure Mode I loading (Fig.4). Similarly, along the length of the specimen, the local principal stress directions are normal and tangential to the inner bore of the specimen. Since the initial crack in the length direction is made along the longitudinal direction, the crack will follow its original direction after it propagates. This self-similar crack growth behavior is confirmed by experimental data. A photo of near tip photoelastic fringes for a machined crack showing the Mode I symmetry is shown in Fig.5.

Stress intensity factor values were determined at maximum depth and along the fin surface. Pertinent test data and results are summarized in Table 1. Normalized values of K denoted as F were normalized with respect to the crack depth a. Since the cracks grew more along the fin surface than in the depth direction, this normalization procedure appeared to produce a contradiction since more crack growth was expected to lower the value of F more than where lesser growth occurred. However, this method was used in order to compare surface and depth values in the same normalized form. If the surface K values were normalized with respect to c (Table 1) rather than a, the contradiction was minimized or removed. The distribution of F, for the part-through cracks, was not uniform along the front of the crack, and the maximum F occurred at the center of the crack front. The variation of F along the crack front was due to the three-dimensional geometric effect on the stress state along the crack front. It is interesting to point out that while some of the part-through cracks penetrated the outer wall in the depth direction, none of the cracks penetrated the length of the cylinder. Also, from Table 1, we note that for the same a/t value the value of F for the long crack is much higher than that for the part-through crack. These results suggest that the practice of using a through-the-cylinder length crack in design may be a substantial overdesign in some cases.

CONCLUSIONS

In this study, the three-dimensional effects on the crack growth behavior in centrally perforated cylindrical specimens with pre-cracks were investigated, using stress frozen techniques. Experimental findings revealed that for the part-through crack the distribution of the Mode I stress intensity was not uniform along the crack front and the cracks grew in a self-similar fashion under pure Mode I loading. It also revealed that for a same a/t ratio the Mode I stress intensity factor for the long crack was much higher than that of the part-through crack, indicating that the practice of using a through-the-cylinder length crack in design might be a substantial overdesign in some cases.

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Table1. Summary of Test Results

	a	c	a/c	a/t	P_{sf}	K	F
Test 3A	15.38	30.74	0.50	0.41	.033	0.31	1.67
Test 5B	14.60	181.65	0.04	0.41	.041	0.74	2.49

$$F = \frac{K}{P_{sf}}\sqrt{\frac{Q}{\pi a}}, \quad Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

Linear Dimensions = mm; Pressure - N/mm^2

$K = N/mm^{3/2}$; P_{sf} = stress freezing pressure

a = crack depth
c = half crack length in the fin surface
t =distance from the fin tip to the outer surface of the cylinder

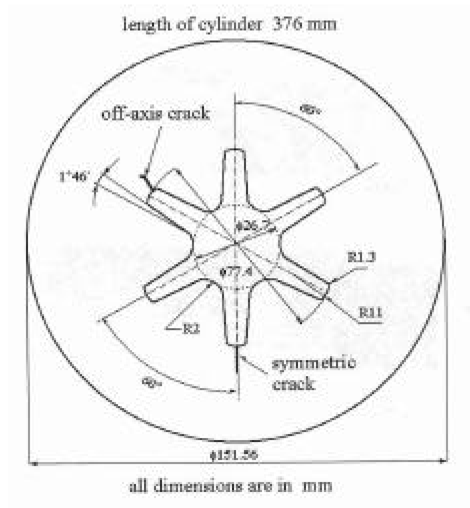

Fig. 1 Specimen Geometry.

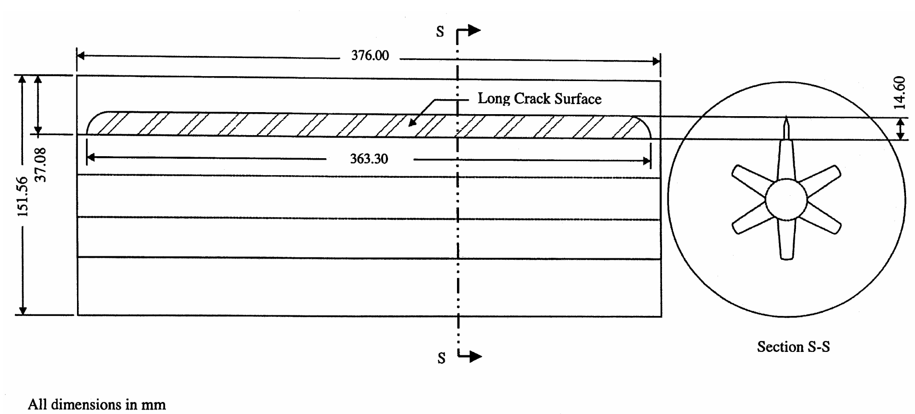

Fig. 2 Geometry of the Specimen with a Long Crack.

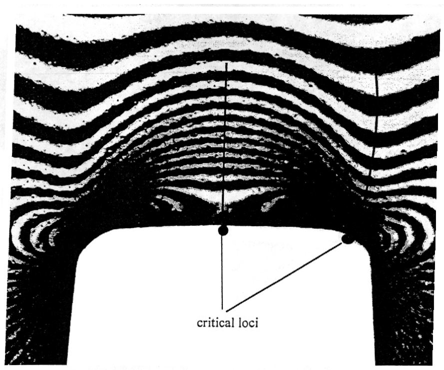

Fig. 3 Fringe Patterns near Critical Loci at the Fin Tip.

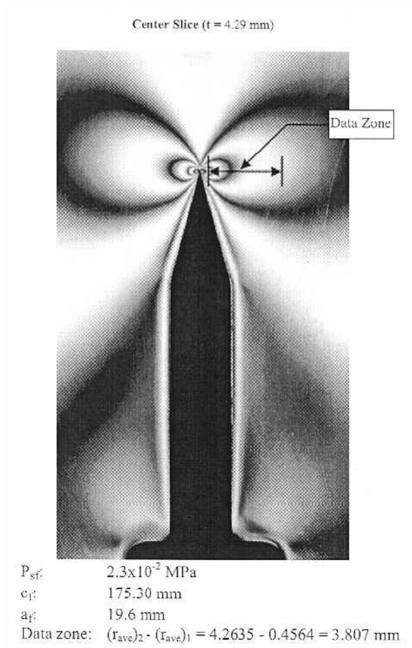

Fig. 4 Fringe Patterns near the Tip of a Machined Symmetric Crack.

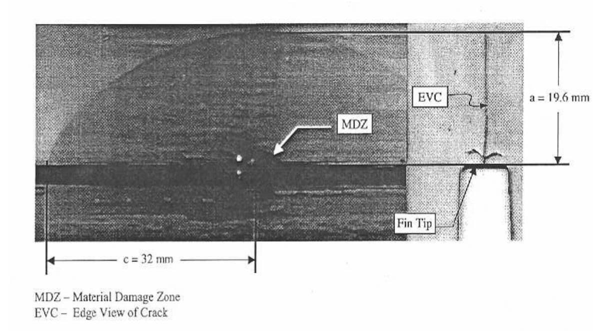

Fig. 5 Plane and Edge Views of Natural Symmetric Crack.